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The Physical Nature of Materials Strengths**

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The strength of a material is assessed most often by means of a tensile test. For a given material with an original cross-section area A_0, if the applied maximum tensile force is equal to $F_{\max}$, the fracture strength can be calculated by $\sigma_F=F_{\max}/A_0$, as described in the textbooks.[1,2] For a bulk metallic glassy specimen, it often fails in a shear mode, as shown in Figure 1, and the shear fracture surface makes an angle of θ_T =56° with respect to the tension axis. Such shear fracture behavior has been widely observed in many metallic glasses, as summarized in the literatures[3,4] and Table 1.[5–14] According to the definition in the textbooks,[1,2] the tensile fracture strength of the metallic glass should be equal to $\sigma_T=F_{\max}/A_0$. However, the actual area of the shear fracture surface becomes $A_0/\sin(\theta_T)$ and the applied normal tensile force on the shear plane is $F_{\max}\cos(\theta_T)$, as shown in Figure 1. This will result in another tensile strength $F_{\max}\sin(\theta_T)\cos(\theta_T)/A_0$, which is different from that ($F_{\max}/A_0$) defined in textbooks.[1,2] Consequently, this gives rise to some interesting and significant questions. Which is the real tensile strength of a metallic glass, $F_{\max}/A_0$ or $F_{\max}\sin(\theta_T)\cos(\theta_T)/A_0$? Why do metallic glasses often fail neither along the maximum normal stress plane (θ_T =90°) nor along the maximum shear stress plane (θ_T =45 °) under tensile loading[5–14]? What is the physical nature of the materials strength?

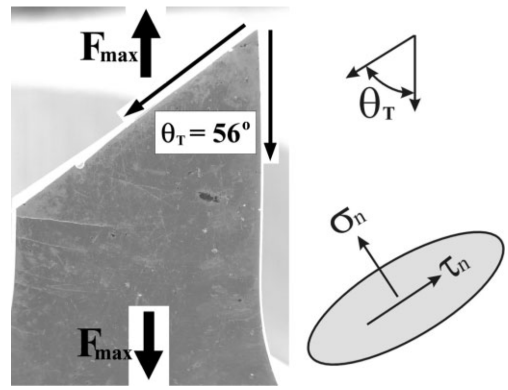

Fig. 1. Macroscopic tensile shear fracture morphology of $Zr_{52.5}Ni_{14.6}Al_{10}Cu_{17.9}Ti_5$ metallic glassy specimen. The nominal tensile fracture strength of the specimen is about 1.58 GPa and the shear fracture surface makes an angle of about 56° with respect to the tensile direction.

For a material subjected to a tensile force F, there is always a combined stress (σ_n,τ_n) on any plane, as illustrated in Figure 2(a). In order to better understand the physical nature of materials strength and answer the interesting questions above, we proposed that there are only two independent intrinsic strengths σ_0 and τ_0 for an isotropic material[4]. As illustrated in Figure 2(b), σ_0 is defined as the critical strength of a material in a Mode I failure; τ_0 is the critical strength of a material in a Mode II fracture. If any plane of a material is subjected to a combined stress (σ_n,τ_n), the tensile failure condition can be expressed by the following criterion[4],

$$(\sigma_n/\sigma_0)^2 + (\tau_n/\tau_0)^2 = 1. \quad (1)$$

Meanwhile, the tensile stress state (σ_n,τ_n) on any shear plane follows the Mohr-circle equation, i.e.

$$(\sigma_n - \sigma_T/2)^2 + (\tau_n)^2 = (\sigma_T/2)^2. \quad (2)$$

According to Equations 1 and 2, the two independent strengths, τ_0 and σ_0 can be derived as:

$$\tau_0 = \sigma_T/2\sqrt{1-\alpha^2}. \quad (3a)$$

$$\sigma_0 = \sigma_T/2\alpha\sqrt{1-\alpha^2}. \quad (3b)$$

Here, σ_T =$F_{\max}/A_0$ is the so-called tensile fracture strength; α is the fracture mode factor[4] and can be calculated from the macroscopic tensile shear fracture angle, see the discussion below. Here, it is suggested that σ_T =$F_{\max}/A_0$ can be only regarded as nominal fracture strength, rather than the intrin-

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Table 1. Comparison of fracture strength, tensile shear fracture angles and intrinsic strengths for different metallic glasses from the references available. The fracture strength and tensile shear fracture angles were obtained by the different investigators for different metallic glasses, then the two intrinsic strengths and ratio were calculated according to the unified fracture criterion.

Investigators	Compositions	$\sigma_T = F_{max}/A_0$ (GPa)	θ_T (degree)	τ_0 (GPa)	σ_0 (GPa)	$\alpha = \tau_0/\sigma_0$
He *et al.* 5	$Zr_{52.5}Ni_{14.6}Al_{10}Cu_{17.9}Ti_5$	1.66	~55	0.96	1.91	0.504
Inoue *et al.* 6	$Cu_{60}Zr_{30}Ti_{10}$	2.00	~54	1.14	2.36	0.485
Lewandowski *et al.* 7	$Zr_{40}Ti_{12}Ni_{9.4}Cu_{12.2}Be_{22}$	1.98	~51.6	1.11	2.44	0.455
Liu *et al.* 8	$Zr_{52.5}Ni_{14.6}Al_{10}Cu_{17.9}Ti_5$	1.65	~54	0.94	1.95	0.485
Mukai *et al.* 9	$Pd_{40}Ni_{40}P_{20}$	1.65	~56	0.97	1.85	0.522
Noskova *et al.* 10	$Co_{70}Si_{15}B_{10}Fe_5$	1.48	~60	0.91	1.57	0.577
Xiao *et al.* 11	$Zr_{52.5}Ni_{10}Al_{10}Cu_{15}Be_{12.5}$	1.75	~55	1.01	1.96	0.504
Zhang *et al.* 12	$Zr_{52.5}Ni_{14.6}Al_{10}Cu_{17.9}Ti_5$	1.66	~56	0.97	1.86	0.522
Zhang *et al.* 13	$Zr_{59}Cu_{20}Al_{10}Ni_8Ti_3$	1.58	~54	0.90	1.86	0.485
Zielinski *et al.* 14	$Ni_{75}Si_8B_{17}$	1.59	~53	0.90	1.93	0.464

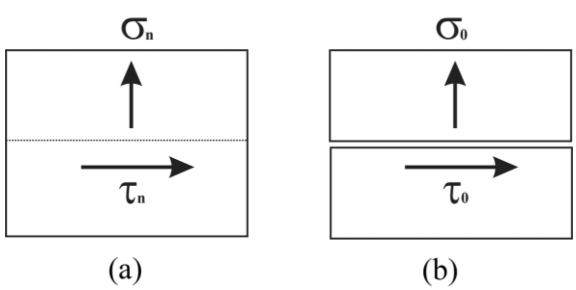

Fig. 2. (a) Illustration of a matter subjected to a combined stress state (σ_n,τ_n) and (b) failure at the two independent intrinsic strengths (σ_0,τ_0)

sic strength of a material. Only when the metallic glass fails along the maximum normal stress plane, i.e. $\theta_T = 90°$, one can get $\sigma_T = \sigma_0$.[4] However, most metallic glasses always fail in a shear mode with different shear fracture angles,[5–14] accordingly, the so-called tensile fracture stress σ_T is not equal to the critical strength σ_0. Besides, the ratio $\alpha = \tau_0/\sigma_0$ is defined as fracture mode factor and is a function of the shear fracture angle θ_T,[4]

$$\alpha = \sqrt{\sin^2\theta_T - \cos^2\theta_T}/\sqrt{2}\sin\theta_T \tag{4}$$

Substituting Equation 4 into Equations 3(a) and 3(b), one can calculate the two independent intrinsic strengths σ_0 and τ_0, which are the functions of the so-called tensile fracture strength $\sigma_T = F_{max}/A_0$ and the shear fracture angle θ_T, i.e.

$$\sigma_0 = \sigma_T \sin^2\theta_T / \sqrt{\sin^2\theta_T - \cos^2\theta_T} \tag{5a}$$

$$\tau_0 = \sigma_T \sin\theta_T/\sqrt{2}\,. \tag{5b}$$

In essence, the strength σ_0 represents the critical stress to break a material in mode I cracking; the strength τ_0 is the critical resistance to overcome mode II shearing of materials. Thus, the ratio $\alpha = \tau_0/\sigma_0$ can also be regarded as an intrinsic parameter of materials, which affects the failure Modes I or II of different materials.

It is well known that the strengths of various metallic materials are significantly different due to the difference in their microstructures in detail.[1,2] Based on the strength analysis above, in principle, it provides a new clue to link the strength relation for a variety of metallic materials with different microstructures from the viewpoint of phenomenology. For ductile single crystals, slip deformation often occurs at a very low critical resolved shear stress τ_0 (e.g. ~ 1 – 10 MPa)[15]. Meanwhile, their strength σ_0 should be high enough because cleavage fracture is extremely difficult to occur in ductile single crystals. Therefore, the ratio $\alpha=\tau_0/\sigma_0$ of various ductile single crystals should be very close to 0. According to the Schmid law and the unified tensile criterion[4], the applied tensile stress for slip deformation can be expressed by

$$\sigma_T = \tau_n/\Omega = \tau_0\sqrt{1-(\sigma_n/\sigma_0)^2}/\Omega. \tag{6}$$

Here, Ω is the Schmid factor of the single crystals, τ_0 is the critical resolved shear on the slip plane (typically (111) plane in FCC crystals). With further plastic deformation, the ductile single crystals often display strain hardening, i.e. the critical resolved shear stress τ_0 increases owing to the multiplication of dislocations within slip bands[16] (see Fig. 3(a)). Besides, there often occur strong interactions between primary and secondary slip bands in the single crystals deformed at high strain level,[17] as shown in Figure 3(b), which can also cause the obvious increase in the shear strength τ_0 due to the lateral hardening mechanism.[15] Another important strengthening mechanism is often caused by dislocation piling-up at grain boundaries in bicrystals[18,19] or polycrystals,[20–22] resulting in a significant increase in the shear strength τ_0 due to the GB

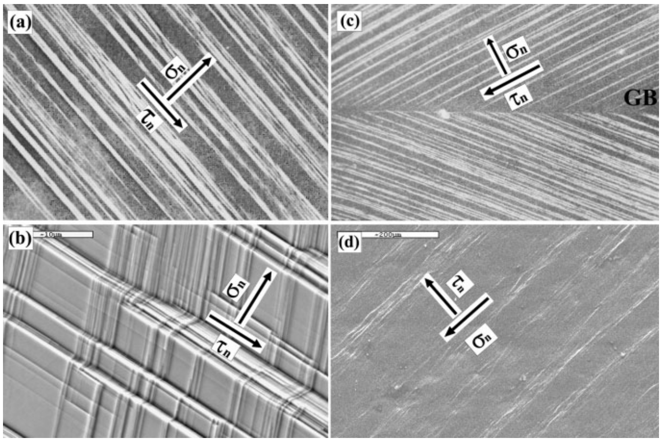

Fig. 3. Slip bands or shear bands in different materials, (a) slip bands in a deformed copper single crystal; (b) interactions of primary and secondary slip bands in a deformed Cu-Al single crystal; (c) impingement of slip bands to grain boundary in a deformed copper bicrystal; (d) shear bands in a deformed ultrafine-grain Al-Cu alloy.

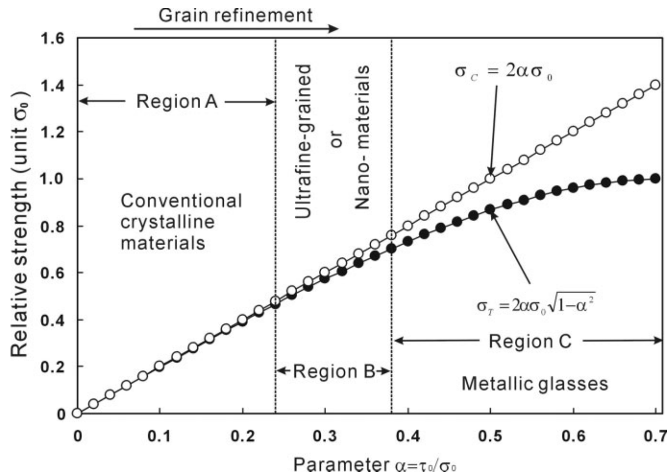

Fig. 4. Dependence of strength asymmetry between compression and tension on the ratio $\alpha=\tau_0/\sigma_0$ with decreasing grain size of the materials.

blocking effect to the slip bands, as shown in Figure 3(c). With great grain refinement, shear bands become the predominant plastic deformation mode in ultrafine-grained or nano-crystalline materials,[23,24] as shown in Figure 3(d). Those materials often possess very high shear strength τ_0 in comparison with their counterparts with coarse grains owing to the refinement strengthening effect.[21–24] The significant increase in the critical shear strength τ_0 will always lead to a high ratio $\alpha=\tau_0/\sigma_0$. For metallic glassy materials, it is naturally considered that the amorphous state is the ultimate limit for grain refinement of crystalline materials, and possess the highest ratio $\alpha=\tau_0/\sigma_0$.

Based on the data available in Table 1, the two intrinsic strengths σ_0 and τ_0 of different metallic glasses were calculated by substituting their nominal tensile fracture stress σ_T and the shear fracture angle θ_T into Equations 5(a) and 5(b). It can be seen that σ_0 is in the range of 1.5 – 2.5 GPa for the metallic glasses in Table 1, and τ_0 is in a level of ~ 1.0 GPa, which is greatly higher than the critical resolved shear stress of various single crystals.[15] Therefore, if considering that all the σ_0 values of the various materials in different states are identical, with increasing shear strength τ_0, the ratio $\alpha=\tau_0/\sigma_0$ should follow an increasing order from single crystals, bicrystals or coarse-grained, conventional crystalline, ultrafine-grained, nano-crystalline materials and metallic glasses.[15–24] Assuming that all the materials yield or fail in a shear mode and their strengths follow the unified criterion[4] under tensile loading and Tresca criterion under compressive loading, respectively, their nominal compressive and tensile strengths σ_C and σ_T can be expressed as,

$$\sigma_C = \tau_0 / (\sin 45° \cos 45°) = 2\,\alpha\sigma_0 \qquad \text{(Tresca criterion)}, \quad (7)$$

and

$$\sigma_T = 2\alpha\sigma_0\sqrt{1-\alpha^2} \qquad \text{(Unified criterion)}. \quad (8)$$

As illustrated in Figure 4, the dependence of the nominal compressive and tensile strengths (σ_C and σ_T) on the ratio $\alpha=\tau_0/\sigma_0$ can be clearly seen. When $\alpha = \tau_0/\sigma_0 \le 0.240$, the two strengths σ_C and σ_T are nearly the same, which can well explain why the conventional crystalline materials with coarse grains are seldom to occur the strength asymmetry under compression and tension (see Region **A** in Fig. 4). With further increasing the ratio $\alpha=\tau_0/\sigma_0$, the strength asymmetry ($\sigma_C-\sigma_T$) is visible, as marked Region **B** in Figure 4. Previously, we have summarized that the ratios $\alpha=\tau_0/\sigma_0$ of various metallic glasses are in the range of 0.385 – 0.707.[4] Under tensile and compressive loadings, metallic glassy materials often display certain strength asymmetry,[3–14,28] which is well consistent with the Region **C** in Figure 4. The different strength asymmetry can be attributed to a relatively high value of the ratio $\alpha = \tau_0/\sigma_0 = 0.385 – 0.707$ according to the unified failure criterion.[4] Between the Regions **A** and **C** in Figure 4, the ratio range of $\alpha = \tau_0/\sigma_0$ is in the range of 0.240 – 0.385, obviously, the strength asymmetry in region **B** is also slight. It is suggested that the ratio $\alpha = \tau_0/\sigma_0$ in Region **B** should correspond to the intrinsic properties of those ultra-fine grained or nano-crystalline materials.[25–27]

In summary, σ_0 and τ_0 can be regarded as two independent intrinsic strengths for an identical material in nature. The testing nominal fracture strengths (σ_T and σ_C) of various materials do not represent their intrinsic strengths, but are only a reflection of the combined effect of the strengths σ_0 and τ_0 under different stress states (σ_n,τ_n). During plastic deformation, all the slip or shear bands of materials are always activated under a combined stress state (σ_n,τ_n), as illustrated in Figure 3(a) – (d). When the coarse-grained materials are gradually refined to fine-, ultrafine-, or nanocrystalline- grained materials,[20–27] even forming amorphous material,[3–14] the ratio $\alpha=\tau_0/\sigma_0$ will be increased continuously. Normally, the nominal compressive strength σ_C is mainly determined by the shear strength τ_0; however, the nominal tensile strength σ_T is controlled by both of strengths σ_0 and τ_0. Therefore, when the materials possess a high ratio of $\alpha = \tau_0/\sigma_0$, it is not difficult to understand why the strength asymmetry (σ_C-τ_T) often occurs in those high-strength materials, typically in metallic glassy materials,[3–14,28] rocks or ceramics,[3] as well as those ultrafine-grained or nano-crystalline materials.[20–27] Furthermore, it is suggested that the proposed new concept about the physical nature of materials strengths is significantly important for better understanding of strengthening mechanisms and optimum design in microstructures for the high-performance materials practical in engineering application.

Experimental

In the current research, different metallic materials were employed, for example pure Cu and Cu-Al single crystals, Cu bicrystals were grown by the Bridgman method in a horizontal furnace,[12,19] ultrafine-grained Al-Cu polycrystals were fabricated by equal channel angular processing (ECAP) technique. Besides, some Zr-, and Ti-based bulk metallic glasses were prepared by arc-melting elemental Zr, Cu, Al, Ni and Ti with a purity of 99.9 % or better in a Ti-gettered argon atmosphere.[12,13,28] For reaching homogeneity, the master alloy ingots were re-melted several times and were subsequently cast into copper molds with different dimensions, i.e. 40 mm × 30 mm × 1.8 mm for tensile test specimens and 3 mm in diameter and a lengthen of 50 mm for the samples used for compressive tests. The compression and the tensile tests of bulk metallic glasses were conducted at different strain rates with an Instron 4466 testing machine at room temperature. Tensile and fatigue tests of the pure Cu and Cu-Al single crystals, Cu bicrystals and ultrafine-grained Al-Cu polycrystals were performed on Shimadzu testing machine. After fracture, all the specimens were investigated by scanning electron microscope (SEM) to reveal the fracture surface morphology and the fracture features.

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